Indian Statistical Institute, Bangalore

B. Math (Hons.) First Year

First Semester - Linear Algebra I

Back Paper Exam — Date: 28th December 2022

Maximum marks: 100 — Duration: 3 hours

1. Prove that number of elements in a linearly independent set is not more than the number of elements in a (finite) basis (**Marks: 14**).

2. Prove that two bases have the same numbe of elements (**Marks: 06**).

3. A linear transformation T is bijective if and only if $T(B)$ is a basis for any basis B (**Marks: 15**).

4. Prove that $\min\{r_0(A), r_0(B)\} \geq r_0(AB) \geq r_0(A) + r_0(B) - n$ for any two square matrices A and B (**Marks: 20**).

5. Prove that there are g-inverses B and C of A such that $r_0(B) = r_0(A)$ and C has full rank (**Marks: 15**).

6. Prove that all g-inverses of A are of the form $A^g + U(I - AA^g) + (I - A^gA)V$ where A^g is a given g-inverse of A (**Marks: 10**).

7. Can $\begin{pmatrix} 0 & 7 & 5 & 12 \\ 2 & 5 & 8 & 11 \\ 1 & 3 & 5 & 7 \\ 0 & 2 & 0 & 3 \end{pmatrix}$ be written as a product of a lower and a upper triangular matrix? Justify your answer (**Marks: 10**).

8. Prove that pivots columns of a upper echelon matrix are independent and span the column space (**Marks: 10**).